AD-A162 353 CONTINUUM STRUCTURE FUNCTIONS(U) STATE UNIV OF NEW YORK AT STONY BROOK DEPT OF APPLIED MATHEMATICS AND STATISTICS L A BAXTER 30 SEP 85 AFOSR-TR-85-1095 1/1

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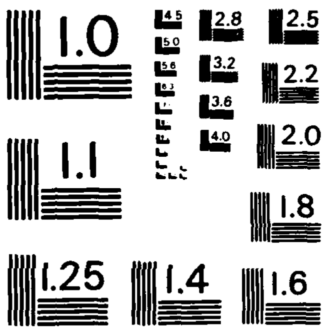

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AD-A162 353

CONTINUUM STRUCTURE FUNCTIONS

First Annual Report to the Air Force Office of Scientific Research, AFSC, USAF.

Grant Number: AFOSR-84-0243

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SUMMARY

A continuum structure function is a nondecreasing mapping from the unit hypercube to the unit interval. The theory of such functions generalizes the traditional theory of binary and multistate structure functions, permitting more realistic and flexible modelling of systems subject to reliability growth, component degradation and partial availability.

During the first year of work on this topic, the PI has developed a theory of modules (i.e. subsystems), calculated various sets of bounds on the distribution of the structure function when the component states are random variables, deduced axiomatic characterizations of two important special cases and derived a definition of the reliability importance of the various components.

1. INTRODUCTION

A continuum structure function (CSF) on the unit hypercube is a mapping $\gamma: [0,1]^n \mapsto [0,1]$ which is nondecreasing in each argument; we assume, without any loss of generality, that $\gamma(\underline{0}) = 0$ and $\gamma(\underline{1}) = 1$, writing $\underline{\alpha} = (\alpha,\ldots,\alpha) \in \Delta = [0,1]^n$. Such functions are used to relate the states $\underline{x}$ of the components C of a machine to that of the machine itself, generalizing the well-known theory of binary and multistate structure functions. In this report, we review the PI's research on such functions during the first year of funding under Grant AFOSR-84-0243. It is first convenient briefly to review previous work on this topic.

The first paper on structure functions whose domain is a continuum is that of Ross (1979) who generalizes the classic IFRA and NBU closure theorems. Block and Savits (1984) derive a topological decomposition for upper semicontinuous CSFs on the non-negative orthant. Let $P_\alpha = \{\underline{x} | \gamma(\underline{x}) \geq \alpha$ whereas $\gamma(\underline{y}) < \alpha$ for all $\underline{y} < \underline{x}\}$ where $\underline{y} < \underline{x}$ means that $\underline{y} \leq \underline{x}$ but that $\underline{y} \neq \underline{x}$. Then Block and Savits (1984) show that

$$\gamma(\underline{x}) = \int_0^\infty \max_{\underline{y} \in P_\alpha} \min_{i \in C} I_{\{x_i \geq y_i\}} d\alpha$$

where I_A is the indicator of A. They also use the sets P_α and $K_\alpha = \{\underline{x} | \gamma(\underline{x}) \leq \alpha$ whereas $\gamma(\underline{y}) > \alpha$ for all $\underline{y} > \underline{x}\}$ to derive bounds on the distribution of $\gamma(\underline{X})$ assuming that the X_i's are associated random variables and that γ is a continuous CSF with compact support.

Baxter (1984) studies properties of two special cases of CSFs on the unit hypercube, viz.

$$\zeta(\underline{x}) = \max_{1 \le r \le p} \min_{i \in P_r} x_i$$

(the Barlow-Wu CSF) and

$$\eta(\underline{x}) = \max_{1 \le r \le p} \prod_{i \in P_r} x_i$$

where $P_1, \ldots, P_p$ are the p minimal path sets of a binary coherent structure function.

Baxter (1986) proposes the following definitions of component relevancy.

Definition

C.1 If $\inf_{i \in C} \sup_{\underline{x} \in \Delta} \mu\{x \mid \gamma(x_i, \underline{x}) < \gamma((x+\varepsilon)_i, \underline{x}) \text{ for all } \varepsilon > 0\} = 1$,

γ is strictly coherent.

C.2 If $\inf_{i \in C} \mu\{x \mid \gamma(x_i, \underline{x}) < \gamma((x+\varepsilon)_i, \underline{x}) \text{ for all } \varepsilon > 0, \text{ for some } \underline{x} \in \Delta\} = 1$,

γ is coherent.

C.3 If $\inf_{i \in C} \sup_{\underline{x} \in \Delta} \{\gamma(1_i, \underline{x}) - \gamma(0_i, \underline{x})\} > 0$, γ is weakly coherent.

In the above, μ denotes Lebesgue measure and $(x_i, \underline{x})$ denotes $(x_1, \ldots, x_{i-1}, x, x_{i+1}, \ldots, x_n)$. It can be shown that C.1 $\Rightarrow$ C.2 $\Rightarrow$ C.3. Extension of these definitions to an arbitrary compact, connected Δ is trivial and extension to an unbounded domain is straightforward. Application to a finite domain involves replacing μ by counting measure and a suitable rescaling, in which case C.1, C.2 and C.3 reduce to Griffith's definitions 2.2(i), (ii) and (iii) for multistate structures (Griffith, 1980).

Baxter (1986) proves that if γ satisfies C.2 and an additional requirement called <u>completeness</u> (essentially that if $\gamma(x_i,\underline{0})$ is strictly increasing, it is necessarily the identity function), then $\gamma(\underline{x}v\underline{y}) = \gamma(\underline{x})v\gamma(\underline{y})$ iff $\gamma(\underline{x}) = \sup_{i\in C} x_i$, thereby generalizing Griffith's Proposition 2.2 to the continuum case.

<u>Definition</u>

Let $\{\phi_\alpha,\ 0<\alpha\leq 1\}$ be a class of binary coherent structure functions such that $\phi_\alpha(\underline{y}_\alpha)$ is a left-continuous and nonincreasing function of α for fixed $\underline{x}$ where $y_{\alpha i}$ is the indicator of $\{x_i\geq\alpha\}$, i=1,2,...,n. If

$$\xi(\underline{x}) \geq \alpha \text{ iff } \phi_\alpha(\underline{y}_\alpha) = 1 \quad (\underline{x}\in\Delta,\ 0<\alpha\leq 1)$$

ξ is said to be a <u>Nativg</u> <u>CSF</u>. (It is so called because it generalizes the "second suggestion" of Natvig (1982) to the continuum case.)

Baxter (1986) studies properties of ξ; in particular, it is shown that ξ is coherent, though not strictly coherent, complete and right-continuous. Its functional form is examined in detail.

In Sections 2, 3, and 4, respectively, we review the PI's work on modules, bounds and axiomatic characterizations; this research was performed jointly with Chul Kim, a former doctoral student.

2. MODULES OF CSFs

Before giving the definition of a module, it is convenient to define minimal path sets of upper simple CSFs on the unit hypercube.

A minimal vector to level $\alpha \in \mathrm{Im}\gamma - \{0\}$ is a vector $\underline{x} \in \Delta$ such that $\gamma(\underline{x}) = \alpha$ whereas $\gamma(\underline{y}) < \alpha$ for all $\underline{y} < \underline{x}$. If $\underline{x}$ is a minimal vector to level α, the corresponding path set to level α is the (nonempty) set $T_\alpha = T_\alpha(\underline{x}) = \{i \in C \mid x_i \neq 0\}$.

Definition (Baxter and Kim, 1985a)

Let $T \subset C$ be nonempty. If T is a path set to level α (for some $\underline{x}$) for all $\alpha \in \mathrm{Im}\gamma - \{0\}$, then T is a minimal path set (MPS) of γ.

This definition generalizes minimal path sets of binary coherent structures to CSFs. MPSs do not necessarily exist for arbitrary CSFs and, if they do exist, they may exhibit undesirable properties. The following definition yields a large class of CSFs for which MPSs exist and are "well-behaved."

Definition (Baxter and Kim, 1985a)

A CSF γ is upper simple if it satisfies the following conditions:

C.1 $P_1 \neq \emptyset$ and $P_1 \subset \{0,1\}^n - \{\underline{0}\}$.

C.2 $\bigcup_{i=1}^{r} T_{1i} = C$ where $T_{11}, \ldots, T_{1r}$ are the r path sets of γ to level 1.

C.3 If T is a path set to level α, then T is also a path set to level $\beta \in \mathrm{Im}\gamma - \{0\}$ for all $\beta \leq \alpha$.

C.4 If T_α is a path set to level $\alpha < 1$, then $T_\alpha \subset T_{1i}$ for some path set T_{1i} to level 1.

If γ is upper simple, it has at least one MPS and is weakly coherent. Further, no proper subset of an MPS is itself an MPS and the union of all MPSs is C.

If Υ is a right-continuous, upper simple CSF, then

$$(2.1) \qquad \Upsilon(\underline{x}) = \max_{1 \leq i \leq r} \gamma(\underline{x}^{T_i}, \underline{0}^{T_i^c})$$

where $T_1,\ldots,T_r$ are the r MPSs of γ and where $\underline{x}^P$ denotes $\{x_i \in \underline{x} \mid i \in P \subset C\}$.

Definition (Baxter and Kim, 1985a)

Suppose that γ is weakly coherent, that $A \subset C$ is nonempty and that there exists a weakly coherent CSF γ_1: $[0,1]^{|A|} \mapsto [0,1]$ and a CSF χ: $[0,1]^{n-|A|+1} \mapsto [0,1]$ such that $\gamma(\underline{x}) = \chi[\gamma_1(\underline{x}^A), \underline{x}^{A^c}]$ for all $\underline{x} \in \Delta$. Then (A,γ_1) is a module of (C,γ) and A is a modular set of (C,γ).

Baxter and Kim (1985a) prove the following theorems, thereby generalizing part of the theory of modules of binary coherent structures (Birnbaum and Esary, 1965) to the continuum case.

Theorem 2.1

Let (A,γ_1) be a module of (C,γ) where γ and γ_1 are both upper simple. Then the MPSs of γ_1 are $A \cap T_1,\ldots,A \cap T_k$ where $T_1,\ldots,T_k$ are those MPSs of γ which intersect A.

Theorem 2.2

Let γ be a right-continuous, upper simple CSF with MPSs $T_1,\ldots,T_r$. Suppose that $A \subset C$ is nonempty and that $T_j \subset A$ for j=1,2,...,k whereas $T_j \cap A = \emptyset$ for j=k+1,...,r. Then there exists a weakly coherent CSF γ_A: $[0,1]^{|A|} \mapsto [0,1]$ such that (A,γ_A) is a module of (C,γ).

Theorem 2.3

Let (A,γ_1) be a module of (C,γ) where γ and γ_1 are both upper simple. Then $(A\cap T)\cup(A^c\cap T')$ is an MPS of γ whenever T and T' are MPSs of γ which intersect A.

Theorem 2.1 generalizes Theorem 4.1 of Birnbaum and Esary (1965) and Theorem 2.2 is a partial converse. Theorem 2.3 generalizes part of the Birnbaum-Esary Test for Modularity; conditions on γ under which the converse to this result holds have yet to be established, and hence the following additional condition is required in the proof of Theorem 2.4 below.

C.5 Suppose that γ is upper simple and that $A \subset C$ is nonempty. If $(A\cap T)\cup(A^c\cap T')$ is an MPS of γ whenever T and T' are MPSs of γ which intersect A, then A is a modular set of γ and the associated CSF is upper simple.

Theorem 2.4 (Three Modules Theorem)

Let γ be an upper simple CSF which satisfies C.5. Suppose that A_1, A_2 and A_3 are disjoint, nonempty subsets of C such that $A_1\cup A_2$ and $A_2\cup A_3$ are modular sets of (C,γ) and the associated CSFs are upper simple. Then A_1, A_2, A_3, $A_1\cup A_3$ and $A_1\cup A_2\cup A_3$ are all modular sets of (C,γ). Further, those MPSs of γ which intersect $A_1\cup A_2\cup A_3$ all intersect each of A_1, A_2 and A_3 or else they intersect exactly one of these sets.

3. BOUNDS

The distribution of $\gamma(\underline{X})$ is hard to evaluate in general since (a) γ may be a quite complicated function (b) the X_i's may be dependent (c) n may be large. It is thus desirable to determine bounds on the distribution of $\gamma(\underline{X})$.

Baxter and Kim (1985b) use decomposition (2.1) to prove the following theorem.

Theorem 3.1

Let γ be a right-continuous, upper simple CSF with minimal path sets $T_1,\ldots,T_r$. Then, if $X_1,\ldots,X_n$ are associated random variables,

$$\max_{1\le i\le r} P\{\gamma(\underline{X}^{T_i},\underline{0}^{T_i^c}) \ge x\} \le P\{\gamma(\underline{X}) \ge x\} \le \coprod_{i=1}^{r} P\{\gamma(\underline{X}^{T_i},\underline{0}^{T_i^c}) \ge x\}$$

for all $x \in \mathbb{R}$.

In a manner analogous to the definitions of MPSs and upper simple CSFs, we can define minimal cut sets (MCSs) and lower simple CSFs. These have corresponding properties; in particular, if γ is a left-continuous, lower simple CSF with MCSs $S_1,\ldots,S_k$, then

$$\gamma(\underline{x}) = \min_{1\le j\le k} \gamma(\underline{x}^{S_j},\underline{1}^{S_j^c}). \tag{3.1}$$

Theorem 3.2

Let γ be a left-continuous, lower simple CSF with minimal cut sets $S_1,\ldots,S_t$. Then, if $X_1,\ldots,X_n$ are associated random variables,

$$\prod_{j=1}^{t} P\{\gamma(\underline{X}^{S_j},\underline{1}^{S_j^c}) \geq x\} \leq P\{\gamma(\underline{X}) \geq x\} \leq \min_{1\leq j\leq t} P\{\gamma(\underline{X}^{S_j},\underline{1}^{S_j^c}) \geq x\}$$

for all $x \in \mathbb{R}$.

Suppose, now, that there is a partition $\{A_1,\ldots,A_N\}$ of nonempty subsets of C such that $(A_1,\gamma_1),\ldots,(A_N,\gamma_N)$ are modules of (C,γ) and $\gamma(\underline{x}) = \chi[\gamma_1(\underline{x}^{A_1}),\ldots,\gamma_N(\underline{x}^{A_N})]$ for all $\underline{x} \in \Delta$. Then we say that (C,γ) admits of a modular decomposition $\{\chi,(A_1,\gamma_1),\ldots,(A_N,\gamma_N)\}$. Baxter and Kim (1985b) show that, in such a case, the bounds of Theorems 3.1 and 3.2 may be improved.

Theorem 3.3

Suppose that γ is a CSF with modular decomposition $\{\chi,(A_1,\gamma_1),\ldots,(A_N,\gamma_N)\}$ and that $\underline{X}$ is a vector of associated random variables. Let $Y_j = \gamma_j(\underline{X}^{A_j})$ for $j=1,2,\ldots,N$.

(i) If γ and χ are both right-continuous and upper simple with minimal path sets $T_1,\ldots,T_r$ and $\mu_1,\ldots,\mu_p$ respectively,

$$\max_{1\leq i\leq r} P\{\gamma(\underline{X}^{T_i},\underline{0}^{T_i^c}) \geq x\} \leq \max_{1\leq i\leq p} P\{\chi(\underline{Y}^{\mu_i},\underline{0}^{\mu_i^c}) \geq x\} \leq P\{\gamma(\underline{X}) \geq x\} \leq$$

$$\coprod_{i=1}^{p} P\{\chi(\underline{Y}^{\mu_i},\underline{0}^{\mu_i^c}) \geq x\} \leq \coprod_{i=1}^{r} P\{\gamma(\underline{X}^{T_i},\underline{0}^{T_i^c}) \geq x\}.$$

(ii) If γ and χ are both left-continuous and lower simple with minimal cut sets $S_1,\ldots,S_t$ and $\nu_1,\ldots,\nu_k$ respectively,

$$\prod_{j=1}^{t} P\{\gamma(\underline{X}^{S_j},\underline{1}^{S_j^c}) \geq x\} \leq \prod_{j=1}^{k} P\{\chi(\underline{Y}^{\nu_j},\underline{1}^{\nu_j^c}) \geq x\} \leq P\{\gamma(\underline{X}) \geq x\} \leq$$

$$\min_{1\leq j\leq k} P\{\chi(\underline{Y}^{\nu_j},\underline{1}^{\nu_j^c}) \geq x\} \leq \min_{1\leq j\leq t} P\{\gamma(\underline{X}^{S_j},\underline{1}^{S_j^c}) \geq x\}.$$

We now consider a different set of bounds based on the sets P_α and K_α.

Definition

Let $\{a_t,\ t\in T\}$ be a collection of real numbers such that $0 \leq a_t \leq 1$ for all $t \in T$. Define $\coprod_{t\in T} a_t = \sup_{S\in\Sigma} \coprod_{t\in S} a_t$ and $\prod_{t\in T} a_t = \inf_{S\in\Sigma} \prod_{t\in S} a_t$ where Σ is the set of all finite subsets of T.

NOTATION: Let $U_\alpha = \{\underline{x}|\gamma(\underline{x})\geq\alpha\}$ and $L_\alpha = \{\underline{x}|\gamma(\underline{x})\leq\alpha\}$.

Theorem 3.4 (Block and Savits, 1984)

Let γ be a CSF and suppose that $\underline{X}$ is a vector of associated random variables.

(i) If U_α is closed,

$$\sup_{\underline{y}\in P_\alpha} P\{\underline{X} \geq \underline{y}\} \leq P\{\gamma(\underline{X})\geq\alpha\} \leq \coprod_{\underline{y}\in P_\alpha} P\{\underline{X} \geq \underline{y}\}.$$

(ii) If L_α is closed,

$$\prod_{\underline{y}\in K_\alpha} P\left(\bigcup_{j=1}^{n}\{X_j>y_j\}\right) \leq P\{\gamma(\underline{X})>\alpha\} \leq \inf_{\underline{y}\in K_\alpha} P\left(\bigcup_{j=1}^{n}\{X_j>y_j\}\right).$$

Baxter and Kim (1985c) show that if γ admits of a modular decomposition, these bounds may be improved as follows.

Theorem 3.5

Let γ be a weakly coherent CSF with continuous modular decomposition $\{\chi,(A_1,\gamma_1),\ldots,(A_N,\gamma_N)\}$ and suppose that $\underline{X}$ is a vector of associated random variables. Let $Z_i = \gamma_i(\underline{X}^{A_i})$, $i=1,2,\ldots,N$.

(i) If U_α is closed,

$$\sup_{\underline{y}\in P_\alpha} P\{\underline{X} \geq \underline{y}\} \leq \sup_{\underline{w}\in\mu_\alpha} P\{\underline{Z} \geq \underline{w}\} \leq P\{\gamma(\underline{X})\geq\alpha\} \leq \coprod_{\underline{w}\in\mu_\alpha} P\{\underline{Z} \geq \underline{w}\} \leq \coprod_{\underline{y}\in P_\alpha} P\{\underline{X} \geq \underline{y}\}.$$

(ii) If L_α is closed,

$$\prod_{\underline{y}\in K_\alpha} P\left(\bigcup_{j=1}^{n}\{X_j>y_j\}\right) \leq \prod_{\underline{w}\in\nu_\alpha} P\left(\bigcup_{j=1}^{N}\{Z_j>w_j\}\right) \leq P\{\gamma(\underline{X})>\alpha\}$$

$$\leq \inf_{\underline{w}\in\nu_\alpha} P\left(\bigcup_{j=1}^{N}\{Z_j>w_j\}\right) \leq \inf_{\underline{y}\in K_\alpha} P\left(\bigcup_{j=1}^{n}\{X_j>y_j\}\right).$$

In the above, we use the notation

$\mu_\alpha = \{\underline{z}\,|\,\chi(\underline{z}) \geq \alpha$ whereas $\chi(\underline{w}) < \alpha$ for all $\underline{w} < \underline{z}\}$

$\nu_\alpha = \{\underline{z}\,|\,\chi(\underline{z}) \leq \alpha$ whereas $\chi(\underline{w}) > \alpha$ for all $\underline{w} > \underline{z}\}$.

4. AXIOMATIC CHARACTERIZATIONS

Kim and Baxter (1985) present axiomatic characterizations of the Barlow-Wu and Natvig CSFs. In particular, they show that γ is a Barlow-Wu CSF if and only if it satisfies the following conditions:

E1: γ is continuous

E2: $P_\alpha \neq \emptyset$ and $P_\alpha \subset \{0,\alpha\}^n$, $0 < \alpha \leq 1$

E3: There is no nonempty open set $A \subset \Delta$ such that γ is constant on A

E4: γ is weakly coherent

and that γ is a Natvig CSF if and only if it satisfies E2 and

E1': γ is right-continuous

E4': For each $i \in C$ and all $\alpha \in (0,1]$, there exists an $\underline{x} \in \Delta$ such that $\gamma(\alpha_i,\underline{x}) \geq \alpha$ whereas $\gamma(\beta_i,\underline{x}) < \alpha$ for all $\beta < \alpha$.

The approach was suggested by the characterizations of Borges and Rodrigues (1983).

In deriving these axiomatizations, Kim and Baxter (1985) prove the following results which are of independent interest.

Proposition 4.1

If γ is a continuous CSF, conditions E2 and

$$E2': K_\alpha \neq \emptyset \text{ and } K_\alpha \subset \{\alpha,1\}^n,\ 0 \leq \alpha < 1$$

are equivalent.

Proposition 4.2

If γ is a CSF which satisfies E1, E2 and E3, then $\gamma(\{0,\alpha\}^n) = \{0,\alpha\}$ for all $\alpha \in [0,1]$.

Proposition 4.3

If γ is a CSF which satisfies E1, E2 and E3, then $P_\alpha = \alpha P_1$ for all $\alpha \in (0,1]$.

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APPENDIX

Papers in Technical Journals

"Modules of Continuum Structures" (with Chul Kim). To appear in Reliability and Quality Control ed. A.P. Basu, publ. North-Holland, Amsterdam.

"Bounding the Stochastic Performance of Continuum Structure Functions I" (with Chul Kim). Tentatively accepted by Journal of Applied Probability.

"Bounding the Stochastic Performance of Continuum Structure Functions II" (with Chul Kim). Submitted to Journal of Applied Probability.

"Axiomatic Characterizations of Continuum Structure Functions" (with Chul Kim). Submitted to Mathematics of Operations Research.

Associated Personnel

Chul Kim--a former doctoral student of the PI. PhD awarded in August 1985 for thesis "Continuum Structure Functions: Modules, Bounds, Axiomatization and Reliability Importance."

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SECURITY CLASSIFICATION OF THIS PAGE

AD-A162 353

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 1095
6a. NAME OF PERFORMING ORGANIZATION State University of New York 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) Stony Brook Long Island, NY 11794	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0243
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.:

11. TITLE (Include Security Classification)
Continuum Structure Functions:

12. PERSONAL AUTHOR(S)
Laurence A. Baxter

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Annual	FROM 7/23/84 TO 7/22/85	9/30/85	14

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Major B. W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5027 22c. OFFICE SYMBOL NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

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